MATHEMATICS TEACHING

THE BULLETIN OF THE

ASSOCIATION for TEACHING AIDS in MATHEMATICS

★

No. 6 — APRIL 1958 2/6

ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

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THE BULLETIN OF THE ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

CONTENTS

No. 6 APRIL, 1958 2/6

This Bulletin appears three times yearly.

EDITORIAL

Dr. B. V. Bowden writing in the December issue of *Technology* tells a fascinating story of the recent developments in computers. As a result of the transition from the methods of the quill pen to the desk machines in use, say, just before the War the speed of calculation was increased by a factor of a few hundred. This increase made contemporary commerce, science and engineering possible. By 1952 machines had been developed which could solve differential equations or invert matrices 500 times as fast as a mathematician with the best pre-war equipment. Since 1952 there has been an improvement by the same factor (500) again. In other words, since 1929 improvements in technique which previously took two thousand years have been achieved twice over.

The computing industry is already one of the biggest in the United States, and last year the computing machinery sold was worth more than all kinds of radio communication equipment and electronic test-gear put together.

Children can easily get the idea, "now that machines can calculate we do not need to learn". Nothing could be farther from the truth. Far from making mathematicians redundant, the revolution brought about by the high-speed computer is producing a bigger demand for them every year. Mathematics is needed to build, maintain and manage the machines, and the part which 'ordinary calculation" can play in running them is well illustrated by the report on page 35 of this bulletin, where we see how a short calculation by a programmer can save hours of needless labour by the machine.

Mathematical techniques are being applied to an ever wider range of problems —from weather forecasting and economics to indexing the writings of Thomas Aquinas, analysing the Dead Sea Scrolls and mechanical translation. Computers will provide a living for many of our pupils when they leave school; but what is more they have taken mathematics into more branches of commerce, industry, intellectual enquiry and cultural life than ever before. The paradox is that the machine is making Mathematics a Humanity—and the challenge is to teach it as such.

It is with these thoughts in mind that we turn to the recent report prepared for the Mathematical Association on *The Teaching of Algebra in Sixth Forms*. Some features of this report are warmly welcome, but others disappoint. It is an excellent summary of the classical manipulative techniques which a Sixth Form course should cover, and most University teachers would be delighted if their pupils came to college having mastered the material here outlined. We welcome also the repeated advice to teachers to improve the background of their own knowledge by reading the excellent textbook by Birkhoff and Maclane. Few textbooks can ever have given more solid assistance to a change of heart than their *Survey of Modern Algebra*. Tactically then, the report is first class; but what of the overall strategy? The report quotes a Cambridge document of about 1920 saying of algebra, "this subject is not suitable for University teaching", and goes on to comment on the changed situation today. The changes in 38 years have been phenomenal, but are the changes in the next 38 likely to be any less great? Yet after this time our present pupils will be in positions of leadership and

responsibility, and today's mathematical education will have been their preparation.

The present shift in algebra is away from emphasis on complicated manipulation towards a more general study of operations and structure. The algebra of the report is still almost entirely the algebra of operations on number, whereas modern algebra is the study of operations of all kinds—many of them not on number at all. Is it too much to hope that we may adopt this attitude in school? The attempt is being made in France, and it is being made in the U.S.A. There is little evidence for the view that the latest parts of mathematics to develop historically are the most difficult for the pupil to grasp. Indeed much modern research in pure mathematics has been directed towards putting the foundations of the subject on a simpler logical basis, and much of the psychological work of Piaget suggests that many of the essential notions of modern algebra (which are regarded as a University study) have to form in the child's mind before he is even ready to undertake the study of number in the primary school. Such topics as the algebra of sets and relations might be taught with profit not merely in the Sixth Form but lower down the school as well. In other countries they are learning how to do this, and unless we learn too we shall be left behind.

Of course such ideas have to be presented in suitable terms in a suitable way. The formal, axiomatic way in which groups, rings and fields are presented to pure mathematicians at University would never do in school. The idea must be presented in terms of concrete applications with a similar structure. And here again we must feel disappointment in the report. Of the multitudinous applications of modern algebra (many already in American textbooks) we find scarcely one. It is surely on points like this that teachers need most guidance. Good methods of manipulation are easy to find in books, and examining boards issue syllabuses in plenty. What is difficult for the ordinary teacher to find are the up-to-date uses to which the subject is being put. These applications are not merely pendants to an abstract theory; they are the very sources of the theories themselves. Ability to pass freely from concrete to abstract and back again is not only the hall-mark of the good engineer and the applied scientist, it is also the most powerful source of inspiration which the pure theorist can have.

Finally, what of the 'duffer'? The duffer is always with us; and he often has an irritating way of turning out not quite the duffer which we had supposed. We are short of scientists, and more and more of the boys who scrape a pass at 'O' level and then try Sixth Form mathematics against what we regard as 'better advice' will be needed to apply mathematics in the concrete situations of science and industry. The traditional, abstract and formal methods of teaching algebra have not had much success with them in the past, so can we hope that they will succeed any better in the future? But we need these boys and girls increasingly, and we have to teach them to think systematically about real-life situations, to appreciate the formal structure of problems, to devise mathematical techniques to describe them, and to interpret the final solutions in everyday terms. The teaching of algebra in Sixth Forms will not be on sound lines until it is doing all of this, and not merely the middle stages.

Mathematics is a Humanity—how are we to teach it as such?

CHILDISH TOPOLOGY

W. M. Brookes

Topics in elementary topology can form an adequate and appropriate basis for learning the nature of geometry in a full mathematical setting.

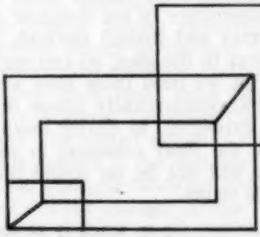
A geometry may be taken as a 'space-model' and features of space may be described by a given geometry appropriate to those features. There is evidence to show that the development of spatial awareness in the growing child takes some years and the possibility of conceptual understanding at, say, eleven or twelve years is limited. A topological approach is possibly much more suitable for the encouragement of 'productive thinking.'

The nature of proof it seems can be more convincingly demonstrated in this field rather than in the field of measurement leading to Euclidean geometries. In the latter case proof is bolstered by the conviction that measurement endows and this militates against a genuine insight. In the early stages of the conventional approach, and for a large number of pupils the later stages too, it is likely that given the necessity of proving that two lines are of equal length, a child has a more positive feeling for his ruler as arbiter rather than for the vague tenuosities of congruence as he knows it.

It would seem that the general confusion that exists between the 'proof' that arises when deductions are made from stated assumptions, and the 'conviction' that arises from a practical action, partly results from the fact that though linear and angular measurement forms the basis of the learning approach to Euclidean geometry the two co-exist at a later stage. That is both theoretical work and measuring work are carried out and it is difficult to draw a line of distinction when the same topics. for example, triangles, are often discussed from these two very different standpoints.

In topological topics proof may be found to be satisfying when the concrete direct action method does not necessarily convince. For example, in the first form of a Grammar School:

(a) What is the least number of strokes of pencil which will be taken to reproduce this pattern without traversing a line twice? (A stroke is a line drawn without taking the pencil from the paper.)



Many will find that it can be done in three strokes but unless one is aware of justification by proof there must remain a doubt as to whether three is the least number.

(b) How many points of crossing are there if 100 straight lines cross each other separately? (I am not going to justify the use of 'straight lines' as a concept except as a basic 'local' notion.)

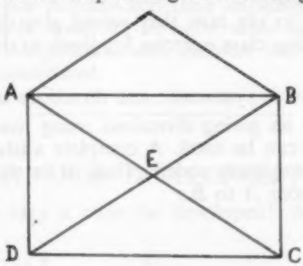
Conviction by counting is impossible—indeed children are often wrong with only six lines. The proof is inductive based on the fact that two lines make one point; hence there are $\frac{1}{2}n(n-1)$ all told.

The development of elementary topological points to the stage of classroom application, that is a course of lessons as opposed to one novelty at the end of term, is not at all advanced. However there is some progress to report and elementary topology takes up roughly a quarter of my first year course at a Grammar School. Among the topics are: (i) Maps (ii) Routes (iii) Junctions (iv) Connectedness (v) Cursive problems in closed networks (vi) Patterns of straight lines, etc.

I shall discuss two of these topics more closely:

(a) *Cursive problems in a closed network*

As has already been suggested the nature of proof as an aid is more easily established if the concrete method is unsure. The oft quoted puzzle of drawing an envelope in one stroke does not keep intelligent children guessing long.



But they are not seeing the structure of the problem. A number of unicursal networks may be presented to them, that is, those with all even or at most two odd junctions. (A junction is a point where three or more lines meet. Thus in the above diagram there are five junctions of which A, B and E are even and D and C are odd.)

Class questions: Can a foolproof method be found to construct the answer?
How many ways can you draw this in one stroke? (A hard question but their concrete activity can lead to insight for some.)
Can you tell me whether this network can be done in one—without using your pencil?

How this develops is a matter of individual teaching, but it will be found that a diagram with two odd junctions has to be quite complicated and disguised to

defeat the drawing efforts of a whole first form. Even so the situation should arise when the concrete method does not give a feeling of satisfaction.

So how can we be sure of the number of strokes?

This kind of work can lead in stages to an awareness of the nature of those networks which can be drawn in one stroke and then to the identification of the number of strokes to draw any given closed network. Then to the defining of the properties of a closed network. Theorems can be developed and we have the beginnings of conceptual insight:

(i) All closed networks have an even number of odd junctions.
(ii) A closed network with $2n$ odd junctions can be drawn with n pencil strokes.
Cor. A closed network with two odd junctions can be drawn with one pencil stroke.
(iii) A closed network with no odd junctions can be drawn in one pencil stroke.

This last is the least fruitful from the learning standpoint, as when approached in a concrete way by drawing it can hardly fail. Start from any junction and the pattern can be drawn in one. Why?

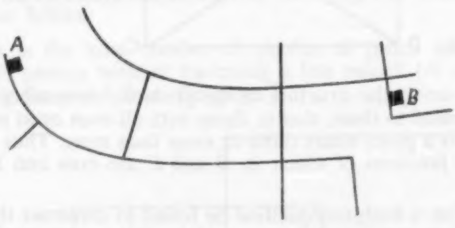
One has to take care not to put the pressure on too severely, but there are enough loose ends to stimulate the brightest and enough steady slogging for the hard worker.

(b) *Routes and route finding*

If children are asked to say how they would give directions to a stranger in their town it is an interesting class exercise for them to discuss each other's efforts and to criticise them.

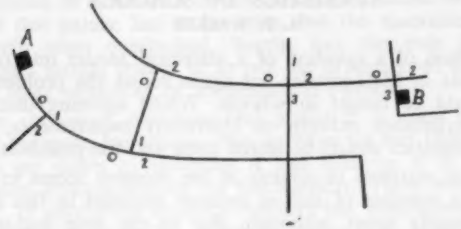
How may routes be made systematic and directions unequivocal?

Exercises may be set on giving directions using junctions only—town maps, 1" O.S. or invented maps can be used. A complete abstraction can be developed where the route can be completely coded. Thus, if in the following diagram it is required to give a route from *A* to *B*.



one may say, using junctions only:

Down the road taking the first turn left.
Turn right at the T-junction.
Cross over the first crossroads and turn right at the next crossroads.
Coded it can be written: *A* 1 1 2 2 3 *B*



Explanation: At each junction label the road on which one stands as 0 and number the branches in a clockwise direction.

Example from an artificial map: Give a route from Abridge to Beachester passing through Seatown but avoiding the bridge at Deeford.

There is a certain amount of artificiality in this topic but there is with the numbering used by the Ministry of Transport and Civil Aviation on the roads in this country. In both cases the notion of distance is omitted and the fact that a useable nomenclature is independent of distance should be emphasised.

The topological work in my classes is linked with 'local' Euclidean geometry by means of the straight line patterns and their implications for 'space modelling.' But there is much more to be done; for instance, ideas using elementary topological notions of congruence in conjunction with metrical notions of congruence remain as possibilities yet to be considered.

STRANGE DEVELOPMENTS

In how many ways may a cube be developed? And what about the other regular solids?

During the last war the British and American forces had developed to an extremely high degree the art of making teaching films, and by the end of the war new training films were being made by the American armed forces at the rate of seven a day. General Keitel, the German Chief of Staff, when interrogated at the end of the war and asked to give his reasons for Germany's defeat, said they had calculated that a certain length of time was needed to bring a German recruit to the stage of efficiency where he could drive a tank or fly an aircraft, they had assumed that the same time would be needed for British and American recruits. That the time had been so much shorter was entirely due to the efficient use of visual aids by the British and American forces.

A. R. MICHAELIS, *Science by Film*,
Aslib Proceedings,
March, 1957.

STATISTICS IN SCHOOLS

A. J. WALKER

The introduction of a question of a statistical nature into certain Advanced Level Examinations in Mathematics has again raised the problem of whether or not statistics should be taught in schools. Whilst agreeing that we should not base our school syllabuses entirely on University requirements, I feel that any new trend in mathematics should be seized upon and the possibilities examined.

The attitude to statistics in schools at the moment seems to be very varied. In schools where a member of staff is suitably qualified in this direction a sixth form course is usually given, although, due to the time factor, the ability to teach the students sufficient to enable them to answer one question of very limited content seems to be the criterion. Where nobody on the staff has had a grounding in statistics, and this seems to be so in an unfortunately large number of cases, there appear to be two alternatives. Either the teacher keeps himself just ahead of the students—a practice which can hardly be recommended—or else he abandons statistics altogether and concentrates on the rest of the examination syllabus.

Statistics is perhaps the first move within the school curriculum which involves something outside the traditional school mathematics. What we learn from the teaching of statistics should be applicable to other branches of mathematics, which must eventually find their way into the schools. This is no wanton speculation for we are now living in a world in which the sciences, particularly applied science, are of increasing importance and the mathematical key must turn in sympathy with our different needs.

Why then do we need to teach statistics? With the ever mounting quantity of research, the increasing complexity of economics, the necessity of knowing the changing needs of populations, and the growing use of automation, is it small wonder that statistics is so important. But then what is statistics? No! it is not just a triad associated with the female figure. No more is it a means of producing results most suitable to one's own purpose. Very briefly it is a completely mathematical science for determining general results from the analysis of random samples; i.e., by considering a few results we can produce an overall picture of a situation. Just as in mathematical analysis when dealing with problems on convergence the value of delta depends upon epsilon, so in statistics does the accuracy of a conclusion depend upon the size of the sample.

Statistics has, I think, had a raw deal on two counts. Firstly there is this attitude that a statistician can always make figures show exactly what he wants them to show, or what his employers want them to show. Besides being unfair to statistics this is unfair to the statistician for, being a mathematician, he must surely be honest with himself. Admittedly many attempts are made to hoodwink the public by various miscalculations, but these could not be made by statisticians, and had we had any statistical training ourselves we would not be taken in by them. In fact, part of the role of statistics is to enable us to uncover doctored evidence, and to stop us from drawing conclusions which though apparently obvious are in reality fallacious. Secondly, much of the apathy which exists towards statistics is brought about by the conception that one is everlastingly

carrying out processes of arithmetic on columns and columns of figures. Granted, a lot of work of this nature has to be done, but the theoretical side is equally important although often overlooked. Surely, too, the ever increasing use of computing machines should enable us to put the drudgery on to one side and to get down to the basic principles of mathematical statistics.

Having then convinced ourselves that statistics is more mathematical and scientific, and of far more importance than perhaps we had hitherto thought, how are we going to get down to accepting it and teaching it in our schools? To begin with we must arrange courses for the teachers. These would not only have the desired effect of producing statistics teachers but would benefit both teachers and pupils in other ways. The contact which this would afford between mathematics teachers from various schools would produce new ideas, discussion and "fresh air" into mathematics teaching, apart from the obvious social benefit. Also the fact that the teacher is still learning is surely satisfying to himself in that it helps to "keep him alive" and in consequence keeps him in touch with the learning situation. Indeed it has often been felt that we teach better whilst learning something ourselves.

Assuming, therefore, that we have obtained teachers qualified in statistics, how should we set about preparing a syllabus? Our first consideration is to decide at what stage statistics should be introduced, since very obviously the content of a sixth form course would be different from a forth or fifth form one. I shall consider two separate syllabuses, one at pre-sixth form level and the other for sixth forms.

Statistics, being a branch of mathematics which is readily adaptable to practical work, is particularly suited to fourth and fifth formers. Unlike experimental work in mechanics, which is usually performed by the teacher as a demonstration, statistics lends itself to practical work by each pupil and the more results that are obtained the better. Furthermore there is no question of "cooking" results to obtain the classical ones, but of "here are your results, what conclusions can we draw from them?" No elaborate apparatus is necessary for these experiments, although ingenious devices can be made if the teacher and class have a particular bent in that direction. Such things as tossing coins, selecting numbers, measuring heights and weight, or sizes of shoes, counting the different makes of cars passing along a road in a certain interval of time, and producing numerous Gallup Polls on different topics are just a few examples of ways in which we can set about obtaining statistical data. At this level the first important task is to discuss the different methods by which we can record the data so that important results can be seen at a glance. Thus we can introduce the histogram and frequency curves. From this stage it does not require much imagination to see the importance of a mode or median and in consequence the even more important mean and standard deviation. Additional refinements may be added if time allows but one must be careful not to confuse the issue with too much professional jargon and too many definitions.

At sixth form level the emphasis should be shifted from the practical side of obtaining and recording data to the more interesting problem of statistical analysis. It is all very well to obtain these means and standard deviations for our

samples but where do we go from there? To answer this we must combine our knowledge of elementary statistics with the theory of mathematical probability. This covers some of the ground normally considered within the realms of pure mathematics, involving such things as permutations and combinations. By comparison of statistical results with the theory of probability we can achieve the real importance of statistics—the prediction of a lot from a little. Further we can consider the Gaussian Curve of Errors—the curve associated with natural variation of physical phenomena and errors produced by sampling and experiment. Thus, for example, we can discuss the theory behind "drawing lines of closest fit" in physics experiments. I do not suggest that any of the analysis should be carried out with the rigour that one normally associates with a college course, but I do feel that in order to make a course in statistics fully comprehensive we must look beyond the fundamentals which I have discussed in connection with the elementary course. Provided that we do not carry things to extremes, I think, that at this level we should consider ways of cutting down arithmetical calculations, and devices such as assumed means should be thoroughly dealt with. Again, if time will allow, an introduction to correlation by way of the "scatter diagram" can easily be made and would afford some more interesting experiments.

At whatever level we introduce children to statistics we must get away from this dreadfully apathetic way of just doing sufficient to enable a stereotyped question to be answered in an examination. Surely it is time that we accepted the fact that we can no longer content ourselves with teaching entirely the old mathematics. It may be argued that we have not got the time to waste on these "new fangled ideas" and nor can we possibly cut anything out of the old syllabus. But is it not better to spend some time on a completely new topic rather than to spend hours of repetition on standard examples of the older mathematics? I am certain that too much practice does not make perfect, in fact that it tends to produce monotony, boredom and perhaps even a sterility of thought.

Whatever ones attitude towards statistics might be I think that we must accept the fact that there are new trends in mathematics and that we must seriously ask ourselves whether we are adequately equipping the coming generation to cope with these new ideas. Statistics is perhaps the first of the more recent developments to reach the schools, but computing and numerical methods cannot be far behind. I am not suggesting any complete change in content of school syllabuses for the present. Much of the older mathematics has a quality for which it would be difficult to find a substitute, but we must recognise these changes and prepare for them accordingly. Even if one supports the idea that mathematics should be taught for its own sake it must be remembered that we have a duty to produce for society the scientists, technicians and workers of tomorrow, as well as the scholars.

What I have said about statistics in schools has been directed mainly at the grammar schools but would apply equally well to technical and modern schools, and as far as the latter are concerned perhaps even more so since they are not usually geared to and impeded by university syllabuses. Now that the universities have made a start by introducing new mathematics into their examinations, let us seize the opportunity of modernising our school courses which have changed little if at all.

FILMSTRIPS IN MATHEMATICS

C. HOPE

In the classroom during a mathematics lesson, various elements combine together to enable a child to structure various concepts, ideas, etc., into relationships and orders by patterns of thinking known as mathematics. We are not concerned with mere instruction designed to convince the child sufficiently for him to accept the teacher's dogmas, but with the child's creative activity by which he discovers and convinces himself and others of the truth and importance of his discoveries. Teaching is positive when it creates problems. The teacher and the child's schoolfellows act on him by stimulating ideas, developing new attitudes, suggesting novel relationships and questioning the reality of the child's assertions about his discoveries. The teacher provides material aids rich in relationships between the elements of a particular field of mathematics so that stimulation shall occur.

In recent times, the film strip has become one of the popular teaching aids. Filmstrips are easily available for use, quickly and conveniently set up and may be used repeatedly being always to hand for revision and retesting. All members of the class participate in exactly the same experience and although the resultant structures and perceptions vary with the individual, there is a common basis for a discussion. A good filmstrip provides a field specially selected for the richness and perhaps the simplicity of its problem content, a challenge to thought and action, not something whose efficiency is tested by pure memory recall. Although the picture content cannot be changed the teacher may vary his vocabulary to suit his children.

In mathematics we use filmstrips for certain distinct purposes. Sometimes we mention great men, Descartes, Newton, Leibnitz; a portrait flashed out on the screen heightens interest and makes the man more real. We talk about a calculating machine. A suitable picture provides sufficient information for the class to deduce the principles by which the machine is constructed. Too many of these pictures used as entertainment destroy the interest they would create.

Sometimes we use a diagram well drawn. For example the graph of $y=(x-1)^2$, with co-ordinate axes and lattice points clearly shown, is projected onto a blackboard. We mark the point $(3, 4)$ in red chalk and challenge the class to explain how the ordinate is calculated (the equation is not given). In the subsequent discussion, the diagram may be altèred, annotated and added to and the chalk may be finally erased leaving the original untouched. The suggestion of raising or lowering the curve poses the problem of its new equation. The next slide confirms the solution which results from the class's discussion.

This technique of projection on to a blackboard or other background of a similar nature may be used in other teaching fields. For example the projected section of a snail's shell enables the teacher to use coloured chalks to emphasis the spiral and its formation. Again Botticelli's *Magnificat* projected onto a blackboard may be more conveniently analysed to show the pentagram which is the basis of its composition.

The traditional filmstrip contains within itself the elements to be structured: it is self sufficient as a source of discussion and information. In mathematics it is possible to construct filmstrips which are tools to be used with other tools to achieve the purpose of stimulating thought in a particular field by making use of the particular circumstances of projection. A filmstrip is projected: therefore all stand back to observe the image without knowledge of its constructor and his indiosyncrasies as might happen were he to draw his diagram on the blackboard. The projector may be moved from side to side conferring a lateral mobility on the picture. The filmstrip housing may be rotated. The two possible motions make possible all orientations of the image on the screen. For example: we draw a circle on the blackboard. We now project the white image of a single straight line on to the blackboard. We move it from side to side, we rotate it so that it is seen to sweep and fill the plane. All this is done from behind or amongst the class; we have no need to go to the blackboard or call for an effort of imagination. The lines fall into two classes, one of lines which do not intersect, the other of lines which do intersect the circle. Discussion and challenge leads us to the conclusion that some lines appear to be members of both sets and we arrive at the notion of tangent as a limiting position. We now add a point outside the circle and move the line so that it passes through the point. All sorts of problems are now possible. The class suggest ones which might be interesting and we proceed from there. In a similar manner another circle projected onto a circle drawn on the blackboard leads to many discussions. Some discussions are fruitful, others end in conclusions seen to be unimportant. We have here all the mobility of the film together with freedom to alter the movement in accordance with the geometrical insights as they are developed by the children in front of us. We have freedom to move and freedom to halt or retrace our steps. There is thus a dynamic quality about this teaching aid completely under the control of the teacher.

This technique may be used where a moving element is superimposed on a fixed field, where a distortion, alteration or movement of one of the parts of the diagram is desired. A filmstrip with these simple tools of elementary mathematics for use with a blackboard would contain squared lattices of various sizes, a line, two lines making an angle, two lines intersecting, two perpendicular lines, one circle, two circles which intersect, two circles which touch, two non-intersecting circles, etc.

It is sometimes useful to have available collections of figures or symbols rich in mathematics which all the class may see for purposes of class discussion: collections which normally would be printed on large sheets or on the blackboard beforehand but more conveniently stored and used in filmstrip form. For example two intersecting sets of concentric circles of integral radii projected are a rich field from perpendicular bisectors to ellipses and hyperbolæ. Choose a law of combination of members of the two sets and various curves follow. The class, stimulated, may draw their own sets and proceed. The blackboard technique comes in useful again in the summary of individual's efforts which follows when the diagram is again projected.

Another example is provided by the array:

```
0 0  0 0 0  0 0
0 0  0 0 0  0 0

0 0  0 0 0  0 0
0 0  0 0 0  0 0
0 0  0 0 0  0 0

0 0  0 0 0  0 0
0 0  0 0 0  0 0
```

This again may yield results of many kinds. One may look at it merely to produce the ways in which 49 may be made or starting with the centre one, adding successive borders we arrive at the sum of

$$1+(4.3-4)+(4.5-4)+\ldots \text{ etc. as } (2n-1)^2.$$

Sucessive frames yield other patterns, each yielding different results or consolidating what has been discovered.

So far we have considered filmstrips as aids to pupil activity stimulating discussion and discoveries. There are teachers who use a filmstrip as a summary of a teaching sequence. A frame is shown for example, the angle subtended at a footballer by the goal mouth. Similar examples are quoted by the class and the strip is then dispensed with as the further possibilities are exploited. The next frame is the next lesson in the sequence. Again suitable collections of fully labelled diagrams are used for revision purposes, e.g., the standard figures for a sequence of geometrical theorems or a sequence of worked examples in algebra. These fail if they give too much information rather than pose the problem which tests the understanding and structuring of the field of revision.

The conditions of projection do not demand complete blackout. Indeed it is an advantage if the teacher, the blackboard and the children can be seen, for not only is class-control easier but tests show that the teaching is more effective when supplemented in this way. Filmstrips as pictures at least prevent complete reliance on monotonous verbalism as well as the mistakes in imagery which can result from verbalism alone.

To sum up, a filmstrip is a teaching aid which of itself cannot compel learning. It is a source of stimulation which the teacher fosters by the use of the right questions, and challenging assertions. It must therefore be rich enough to stimulate this mathematical activity. Throughout, it is the mental activity of the child which will lead him to develop insights and an understanding of the mathematics which is studied: there is no substitute for this activity, only learning aids which seek to remove the obstacles of extraneous factors interfering with the development of the resultant insights. Filmstrips like many other aids can be both obstacle and catalyst; it is important that the resultant reactions be desirable.

IN THE NEWS—II

JOHN V. TRIVETT

In the last Bulletin we saw how much practice in Arithmetic could be had in an interesting way by using a newspaper as a centre of interest, as well as some insight into the comparative incidence of various topics in daily papers. Here are suggestions for another line of development.

Which letter occurs most often in a newspaper? How do you know? How could we find out? Is it necessary to count all the letters? If not, how many should we count?

Each child in the class, with a paragraph chosen somewhere from his newspaper, keeps his own record by listing the letters of the alphabet in a vertical column in the notebook. As each letter of the words, in order, is read, a straight tick is placed in the appropriate row in the record. Gradually, the pupil sees the distribution growing as he works through successive paragraphs, recording as he goes. When four ticks have been placed against any letter, the fifth tick can cross them out, giving a 'five-bar gate' motif which, when repeated often, is far easier to count in totalling.

With the whole class at work, the number of letters counted will be in the thousands in quite a short time. At intervals the teacher can halt everyone and announce that a total is to be taken. At this, each pupil draws a vertical line in his notebook, and records after it, on each line, the total number of letters from the beginning of the count. The first totals may be taken after only a few minutes' work, to give perhaps as few as fifty letters distributed over the whole class; a few minutes later the second total will give some hundreds. Subsequently, more time may elapse between successive totals as the combined distribution grows into the thousands.

While the letter-recording is continuing some pupils can be assigned to collect individual totals by walking round the class and noting on previously-prepared score sheets the pupils' totals. Provided each pupil writes his letter-totals clearly in columns and each total-collector does his job systematically there will be no muddle. The simplest arrangement is for one child to collect the totals for one letter at a time. This work is in itself a grand experience of group work, not to mention one of class management for the teacher!

This continues until ten or twenty thousand letters in all have been counted and recorded. The work is eased by the fact that a letter-counter may select any part of any newspaper, start where he likes, throw the paper away when he is bored with looking at the same piece and start elsewhere. If he loses his place, no matter—it makes very little difference in the long run.

Block graphs are then drawn by everyone—or the work is apportioned out to groups—based on the distribution at successive totals. That is to say, a graph may be drawn first of the distribution of 50 letters, the heights of the 26 'blocks' being relative to the frequency of the letters. Another graph is drawn for the 100-distribution and so on. It is useful to arrange the scales so that the overall sizes of the graphs are about equal: they can then be compared easily.

The pupils are asked to say what they see about the general shapes of succes-

sive graphs, particularly after the first few which may, of course, be very peculiar with so few letters involved and an unusual number of strange letter combinations. Are the pupils' separate graphs of similar shapes? What happens to the general shape as the total of either the individual counts or the class count increases? Ultimately the pupils will not fail to notice that once two or three thousand letters are considered the shape of the graph 'settles down' and by the time 10,000 is reached the shape is constant enough to show very little change subsequently.

From the graphs we are able, therefore, to say how many letters need be counted to tell us fairly accurately which letter occurs most frequently, and the order of frequency of all the other letters. Questions like this now arise:

Would we expect any great difference in graph shape, or distribution, for a million letters? 10 million?

What about newspapers in foreign languages?

How many children need I ask 'Which newspaper do you take at home?' before I know the approximate truth for all the children in the school?

How many children in the school should I weigh to get the average weight of *all* the children?

Discussion of the latter type of question involves the principle of sampling. How shall I be sure that the relatively few children I ask, or weigh, will not be in fact, perhaps unknown to me, an unusual selection who will bias my estimate too much? For school purposes reasonable decisions about choosing children from different classes, junior and senior, boys and girls, will probably suffice though it is fun to do it by means of a random sample, too.

One method is to get a table of random numbers. If this is difficult one can be made by some random selection of numbers from a telephone directory: one pupil chooses a page, another a line, another the position of the figure in the 'number'. This figure can then constitute the first figure of an admission number. When four figures have been so chosen the child's name is located in the admission register against the four-figure number, and this child is admitted as one of the sample.

Different sized samples should be selected and their information compared with each other and with the results for the whole population if this is available. Discussion follows as to the reliability of any assessment made from a sample for the whole of the school. From this, appreciation follows of the now frequent Opinion Polls published in the press, their compilation, and their influence.

Returning to our newspaper letter-count, how can we use the results of the research? The teacher explains the compilation of a printer's font and wonders whether the numbers of letters sold in such fonts bear any relation to the frequency discovered by the class. Someone borrows a font or visits a printer and records the information. Both sets of letters are placed in frequency order commencing with the most common and rank scores are assigned:

letter	Pupils' order	Font order	d=difference	d^2
e	1	1	0	0
t	2	$2\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
a	3	$2\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
o	4	5	1	1

and so on.

(Letters of equal frequency in the font may each be ranked as the mean average. Thus if six letters occupy positions 13th, 14th to 18th, and there is the same number of each, then each scores $15\frac{1}{2}$).

It must now be explained that a connection can be worked out between two such orders by calculating what is called the 'correlation coefficient'. This provides a measure between the two orders. It is obtained by using the formula:

$$\text{corr. coeff.} = 1 - 6 \times (\text{sum of } d^2) / n\ (n^2 - 1)$$

where n is the number of elements in the comparison, 26 in this example, and 'sum of d^2' is the total of column 5 in the above table. Since we are squaring the differences no notice need be taken of any negative differences which may have been obtained. It is seen that if the two orders are exactly the same, $d^2 = 0$, and the coefficient is 1; if the second order is exactly the reverse of the first the coefficient is -1, and the nearer to 0 the coefficient the less there is any correlation.

The pupils work out the d's, calculate the d^2 total, substitute and find the coefficient; they discover with satisfaction that it is nearly 1. This sets a seal of officialness on their discoveries and gives a reason for the printer's font being as it is.

If a visit is made to a printing works or a newspaper office a linotype machine should be inspected as to the frequency of use of the various letters, an order recorded and a similar correlation coefficient calculated. The interest here lies in the fact that a principle different from that of the font is used, though the letter-frequency idea is invariant.

The Morse Code also lends itself to treatment in this way. The code is 'scored' by awarding 1 point for a dot, 3 for a dash, and 1 for a space between. Thus:

A (. —) scores 5
B (— . . .) scores 9, and so on.

This done, the order is obtained with the lowest score in first rank: this will be E=1, obviously. As before, the correlation coefficient is calculated, a high result being obtained again, showing that Mr. Morse was no fool and worked out his code on scientific lines! If preferred a rough and ready comparison can be seen between the two orders if, when the letters are arranged vertically, lines are drawn joining similar letters. The less the lines cross the closer are the two orders.

As written, this survey of a newspaper suggests a definite line of progress but this need not be so. The order and emphasis should vary, dependent on the interests and suggestions of the group of pupils. What has been written is to suggest the wealth of work which can arise and there is yet more: the sizes of print, the mathematical relations inherent in the enlarging and diminishing processes for the reproduction of pictures. Neither is there need to work through it all exhaustively: opportunity can be taken of developing more work on Normal Distributions whether it be to examine population figures or to ascertain the average number of petals on a common daisy. Visits to docks may precipitate enquiries into the sampling of cargoes. It is practically certain that once a class engages successfully in this type of work many other suggestions will arise.

MATHEMATICS IN THE SECONDARY SCHOOL CURRICULUM

IV—MATHEMATICS HAS A UTILITARIAN VALUE

R. H. COLLINS

It should be made clear at the outset that there are varying interpretations of the word 'utility' as used in this context, and it is important to know that in this article the claim that mathematics has utilitarian value is taken to mean that the items taught in school will find frequent use in the daily life of all members of the community; for example, that the teaching of the topics of decimilisation of money, or equations in algebra are valuable because they are techniques required with fair frequency by most members of the community.

This claim is another of long standing and one which has had a curious way of being first in, and then out of favour, right from earliest times.

There is much evidence to show that the Babylonians and Egyptians had only time for the teaching of mathematics so long as it was of practical use. The Greek period, however, was quite the reverse and Plato pleads with his readers "to take hold of it, not as amateurs, but to follow it up until they attain to the contemplation of the nature of number, not for the purpose of buying and selling as if they were preparing to be merchants or husksters, but for the uses of war."

It might be thought that whilst the first part of the statement denies the utility of the study, the second part does the reverse. It should, however, be remembered that Plato is referring to the education of a minority, "soldier and philosopher in one, who are to be the guardians of the State." In this sense, he is, of course, considering the subject to be strongly vocational to the few and yet at the same time it would appear that he also considered the subject to be of great utility to the members of the trading community. In his time, moreover, Plato had only to consider his aims in relation to an educational system for the few—the rich ruling classes—and it is interesting to conjecture what he would have said if he was alive to-day, when there is universal education for a society in which all of us must do some buying and selling as a matter of course. He emphasises the narrow vocational utility for the few when he is dealing later with geometry, for "so much of it applies to the conduct of war, for in dealing with encampments and the occupation of strong places and the bringing of troops into line and column and all the other formations of an army in a battle and on the march, an officer who had studied geometry would be a different person from what he would be if he had not. But for such purposes a slight modicum of geometry and calculation would suffice."

The Roman outlook on the same point appears to have been quite the reverse for Quintilian advocates that "music should be studied largely to help with gestures and train the voice, geometry to aid in settling law suits relating to land, dialetic to aid in detecting fallacies and astronomy to understand the movements of the heavenly bodies and the references of literary writers."

A great influence underlying this change in outlook must have been the intensely practical outlook of the Romans who centred their attention on the material world and ignored the ideal. It is likely that such a mode stemmed from the Greek philosophy of the Sophists rather than that of Plato.

As we come nearer to our own times, the utilitarian aspect of mathematics, kept alive from Roman times, obtained a stronger hold, through the work and writings of the new philosophers who had begun to explore the universe as one based

on measurements (in contrast to one based on judgments) with the consequent division of philosophy into regions of natural science and of religion.

Now the thoughts on the utility of the subject became confused. For whilst the Greek conception of utility may have been valid, later developments in mathematics had lead to the evolution of the special application of the subject as distinct from the universal. As Wormell puts it in *Teaching and Organisation* (1897), "The practical uses of mathematics, skill in calculating, evaluating and measuring, are always in demand. The actuary, the engineer, the architect, the surveyor and valuer and the many others, all live by using it." This writer is under the impression that by mentioning a few highly skilled occupations he is thereby proving the validity of his claim for the study as a 'must' in a state of universal education.

In fairness to the author of this quotation it must be conceded that his outlook was conditioned by association with a mathematical education which was the prerogative of the few, in fact for those who came from families who highly prized careers in the professions mentioned, and in that context the claim of utility does have some validity. But in the wider sense of modern education the point has not been made by Wormell.

Apart from the reference above, on the value of mathematics as a 'utility' subject, I have found little attempt to seriously support the claim by detailed argument. It is more than likely that at quite an early stage in the last century pressure from other subjects for inclusion in the school curriculum will have lead to the 'soft pedalling' of this argument for fear of losing its hitherto undisputed place in the education of the young citizen. Certainly the *Quarterly Journal of Education* Vol. 11 (1831) gives a clue to the way this attack was directed against mathematics.

"It is not sufficient argument for the introduction of such pursuits that their practical applications are of the highest utility to the public and profitable to those who adopt them as a profession. The same holds true of law, medicine, and architecture, which nevertheless find no place amongst the studies of the young. It is considered enough that the lawyer should commence his legal pursuits when his education on other respects is complete."

The word 'utility' can be used in reference to two quite different sets of conditions. In the first a writer might have in mind that the subject would be in constant use in the after school life of the child no matter what career was taken up. On the other hand it might be taken as referring to the applications of the subject to other subjects of the curriculum. To this latter meaning the following quotation from "The Special Reports on Education Subjects," Vol. 26, *The Teaching of Mathematics in U.K.* Part 1 (1912) clearly gives the answer: "A discussion of the value of geometry as a general educational subject, which depends on its relationship to other subjects, only becomes valid when the claims of these other subjects to a universal place in education has been admitted."

If the claim of general utility in after life was meant seriously it ought to have been the prime consideration of those advocating it to try to establish a norm for the amount of mathematics required. Little appears to have been done on this until 1926 when Guy M. Wilson carried out a survey on *What Arithmetic shall we teach?* According to him the processes least needed were apothecaries' measure, partial payments, square root, decimals, compound interest, proportion and much of mensuration, and it is interesting to note that the elimination of some of these topics from G.C.E. syllabuses has only just come about, *viz.*, compound interest,

finding square roots numerically, apothecaries' measures.

Since the study made by Wilson, others in America have also made similar attempts to investigate the amount of mathematics used by the general body of adults in everyday life. Unfortunately, little that is conclusive can be drawn from them.

Bobbitt in his book *How to make a Curriculum* has a chapter on the topic which is based on the assumption that the amount of general mathematics used in everyday life is reflected in the printed thoughts of the world's intellectual leaders. An analysis of scientific articles in five general magazines and three books on popular science was then made and the conclusion reached on algebra was that it consisted chiefly of processes and skills and has very little content and a meagre vocabulary. On the other hand he found much need of "form" in geometry.

The weakness in this kind of approach to the problem lies in its underlying assumption. I doubt if the majority of people do read the kind of literature referred to and until this point can be proven I shall doubt the worthwhileness of the contentions above.

Another means of trying to evaluate the amount of mathematics required by persons in general is through a study of the contents of the daily newspapers. One such survey recently made revealed a remarkably small amount of mathematical content, so much so that it could all well be covered for the vast majority of children by the end of their primary school days, because it was mainly related to basic arithmetic.

THE PRINTED WORD

C. Hope

How many living mathematicians can you name who are actively engaged in research on theory of numbers, algebra, geometry, analysis or some other part of mathematics? Now how many persons can you name who have written text-books on the same topics? This simple test will clearly show the hold that text-books have on our mathematical education. For the past forty years at least, we have grown to depend on the text-book for all our mathematical knowledge, to rely on it for guidance, authority and support. "Let me have your syllabus," says the Head. Immediately we consult the printed oracle to compile a list of topics. There before us is a scheme of work neatly chaptered into convenient topics; nicely exampled with just too many examples for the time it suggests; topics devised and arranged to conform to the terms of each year of the course. We move from page to page, from chapter to chapter, sublimely confident that the standard is adequate, that the contents are mathematics and that progress thereby is inevitable.

Now before you hasten to contradict me, to tell me forcibly that you do not do this, think very carefully. Many of our reviewers of recently submitted books imply that the publication fails because it does not in fact supply the needs I outline above. Does it matter if a figure is drawn in any particular way, if there are too few examples, if there is no practical work, no historical interest? Text-books are just as much teaching aids as geoboards, Cuisenaire material, filmstrips or any other of the devices which adorn the modern classroom during the teaching of

mathematics; the text-book is another tool in the hands of the teacher. If text-books are written in certain ways, it is because the general demand makes them economically successful in that form. People will not buy the books if they do not perform certain functions. It is true that many text-books are written to sell in sets of thirty or forty with an implication that the teacher is merely an informed commentator: the tragedy is that some of us appear to accept the implication as a necessary one.

What do we want in a textbook? If we had a state syllabus this question would be easy to answer, for all would do the same mathematics at the same time and stage. The influence of public examinations has almost achieved this result; textbooks are very similar, conforming to a pattern so that even the diagrams are identical in many cases. Those of us who are able to choose whatever we want to teach, are always meeting the plaintive cry " . . . but what standard are you aiming at . . .?" and being assailed at every turn to accept a particular rung of a particular ladder of examinations as the criterion of the suitability of our choice of topic. Mathematics teaching is thus assailed from both flanks: on the one hand the textbook, on the other the public examination and both work together to defeat the teacher who would go forward. The examination provides the framework which justifies the uniformity of the textbooks.

What do we want in a textbook? The mathematics which we can help children to learn can be largely determined by the children themselves. We wish to present them with some field of real elements or of ideas which will present many problems of a mathematical nature; the field must provide both variety and opportunities for choice on the part of the pupils. We require of our children that they be able to recognise the existence of a problem, to formulate it in such terms as will suggest a possible solution and finally to arrive at a solution, if need be with the assistance of the teacher. There is no natural preordained order in topics as a general rule but it is often convenient to develop certain skills and areas of knowledge before others. The wise teacher chooses his fields for investigation with this in view and by his choice, to a large extent determines the shape of his course. At certain stages, repetition of certain types of problem is necessary to develop and ensure a confidence that success is the inevitable outcome of certain procedures and sometimes to make the exercise of certain procedures automatic. Is the provision of these drill examples the sole contribution that textbooks can make? Textbooks usually link the examples together with what we call 'book-work.' The function of such theory is apparently to summarise the methods by which the next set of examples may be solved or to help a pupil to reassemble a principle or formula now forgotten. To some teachers, it may be, that this serves as a guide to the next piece of mathematics to be attempted.

The present shortage of mathematics teachers inevitably involves teachers, who know little more than the area of knowledge they are to teach, in the mathematical education of children. If these are to use a textbook, then it is necessary that the book-work shall at least force the reader to recognise that a problem interesting enough to solve at that stage exists and to provide opportunity for attempting a solution before a ready-made effort tailored to the needs of certain examination topics is supplied. One would like to see more reference to things to make, experiments to carry out, fields of mathematics to investigate. One would like problems

included which would provide an opportunity for reference to and natural explanation of their solutions' reflection of more advanced mathematics or of modern topics of mathematical investigation, *e.g.*, algebra as a theory of structure, new number fields, geometries without measures, etc., general methods of solution, etc. The present emphasis on the historical setting of a mathematical topic and its influence on the development and application to the conduct of human affairs seems to call for more reading matter, more reference material and more suggestions for further reading. There are many who would say that the function of a textbook is to provide an easy path to examination success and that anything which interferes with its smooth progress or causes it to deviate should be omitted. It may well be that text-books are financially successful when so constructed but the need for supplementary material must be met in some way. Could not a textbook series have included in separate covers some informational material to provide a background for each volume? I do not mean bigger and better teacher's books but something for the pupil to read.

If the change in textbook construction is to be brought about, then some pressure must be exerted first by the teachers themselves on the publishers representatives, second by comment in the educational Press and finally by reviewers. A good review is partly an appraisal of the fulfillment of the author's avowed purpose, partly an appreciation of that purpose in the wider context of the philosophies of education and partly a view of the book's usefulness as a tool in the hands of a teacher. A book may be damned for its principles but praised for its possible uses. One reads a review to find if the book will fit one's present purpose or if it offers any guidance of the treatment of new ideas as to the selection or arrangement of the mathematical curriculum. One selects a textbook in most cases as a second best to the one one ought to write for one's self and having in mind the need for economy one series is chosen. In consequence, the mathematical ideas of a given pupil may be wholly influenced by those of a given author, an increasing danger in these days of alternative syllabuses and textbooks on general mathematics. In the secondary modern and primary schools where the reliance on textbooks is carried to the extreme and where the textbooks often appear to have no concern with mathematics, one series used in a school may prove a menace to a pupil's mathematical development. To counteract this real danger, let us learn to use two or three sets of textbooks taking from each that which inspires, selecting each to fulfill the varied aims we all avow and showing the courage to use an advanced level text to further the experience of first year secondary pupils.

It is customary in scientific writing to state the important points just as often as the unimportant ones: exactly once; and to assume thereafter that this vital information is graven in the minds of all readers. This is one of the wonders of science, for it is economical of space in the journals and it encourages researchers to write up their findings. It also ensures a certain permanence to the work, for it will be years before it is completely understood.

J. D. WILLIAMS, *The Compleat Strategyst*.

MATHEMATICS IN HONG KONG

Chinese students need little or no persuasion to work at their mathematics. The quiet contemplation of figures and characters appears to be part of their cultural heritage.

Nearly half the secondary school population are taught in English and the usual difficulties of learning in a second language are in evidence. The fact that competence in mathematics is not so dependent on the standard of English as it is with the other school subjects is perhaps another factor contributing to its popularity.

Many students are taking the London G.C.E. examinations as well as the Hong Kong School Certificate and University Matriculation examinations. The reason for this is that passes in G.C.E. are recognized as equivalent to the Hong Kong examinations so that the G.C.E. provides a second chance to pass and probably a better chance for it is generally agreed that the Hong Kong Matriculation standard is above that of Advanced level G.C.E. However, this high standard is mainly in mechanical manipulation. Topics like 'imaginary numbers' are included in the curriculum but little stress is laid on its application to practical problems.

These high standards mean that in the University only a very few potential Honours mathematicians are trained. As a result schools are likely to find a serious shortage of ordinary degree-trained staff when those who came as refugees from the Chinese mainland have to be replaced.

Few, if any, schools take mechanics at Ordinary level but in response to University Faculty entrance requirements Applied mathematics is slowly being introduced in the Sixth Form.

In the secondary schools promotion is according to achievement so that students in all the fourth forms for instance are expected to pass a common examination before promotion to the fifth. The content of the curriculum and the depth to which it is studied is not adapted to any appreciable extent to the pupil's individual needs. There is also a tendency to work steadily through the textbook from cover to cover.

Teaching aids are not widely employed but it augers well for the future that the recently appointed Visual Aids Officer is himself a mathematician.

Altogether teachers from the United Kingdom enjoy teaching the Chinese students. Staff are well provided for and mathematicians are at present needed to fill the Government Overseas Civil Service appointments.

R. F. SIMPSON.

MAKING MONEY

I am in charge of twenty machines which are minting pennies. One of these machines is not working properly and is making all of its pennies the wrong weight by some constant amount. I know that a penny should weigh one third of an ounce, and I have an accurate, graduated spring balance which is capable of weighing a number of pennies at once. How can I discover the faulty machine and the error in weight of the coins which it produces by making *only two* weighings on the balance?

How can we best modify the problem if the balance is not calibrated?

THE HISTORICAL BACKGROUND

C. Birtwistle

The appeal which any subject makes to an individual varies considerably with the individual's interests. In the successful teaching of any subject, therefore, one should take advantage of these different interests to arouse the interest of the pupil in that particular subject. One such approach which is often overlooked by teachers of mathematics is the appeal to the pupil who is particularly interested in people and events of the past.

The historical approach to a topic gives added interest, and the mention of the way in which a particular mathematical problem was tackled in the past often gives an insight into the reason for current methods, or some idea of the way mathematical knowledge has been built up. For example, when first introducing the graph of a quadratic function as being a parabola, one might mention the conic sections, the investigation of their properties by the Greeks and the subsequent use of these properties by Kepler.

In the Mathematics Room wall displays can often be arranged to feature famous mathematicians or to pose some mathematical problem which has occupied the labours of mathematicians in the past (*e.g.*, the Konigsberg Bridges problem, or the trisection of an angle).

A source of portraits of mathematicians is the two portfolios published by Scripta Mathematica of New York, 'Portraits of Eminent Mathematicians', which contain some twenty-five portraits each in a folder containing a brief life history of the individual. The portfolios cost about £2 each, but for those who find this too costly an admirable portrait gallery of famous mathematicians may be obtained on postage stamps for a few shillings. Details of these with the country of publication are as follows: Copernicus (Poland), Tycho Brahe (Denmark), Galileo (two studies) (Italy), Pascal (France), Huygens (Holland), Descartes (France), Romer (Denmark), Leibnitz (Germany), Bolyai (Hungary), Abel (Norway), Hamilton (Ireland), Chebishef (U.S.S.R.), Lorentz (Holland). Details may be obtained from any stamp catalogue.

For reference purposes on the history of the subject, the standard work is F. Cajori: *A History of Mathematics* (Macmillan). There are, however, a number of less expensive and very readable accounts. Vera Sandford's *A Short History of Mathematics* (Harrap) and the more recent *A Concise History of Mathematics* by D. J. Struik (Bell) are both well illustrated and form excellent reading. *Men of Mathematics* by E. T. Bell is now generally accepted as one of the best books on the subject and was recently re-published by Pelican Books in two volumes. *The Great Mathematicians* by H. W. Turnbull (Methuen) compresses a wealth of detail into a mere 130 pages and holds one's interest throughout. A. Hooper's *Makers of Mathematics* (Faber) is another book giving a good general approach to the subject.

In dealing with the history of mathematics, local history should not be forgotten. If one is interested in local history it is an easy matter to find some local individuals of the past who had some interest in mathematics and to present a 'potted biography' for display on the Mathematics Room wall; the detail one uncovers is often an education for oneself! For example, for the little series which I have in my own Mathematics Room, I have found particulars of a number of

distinguished men with mathematical interests and activities, and who lived within five miles of the School, ranging from one who built and equipped the Royal Observatory at Greenwich to a namesake of mine who published work on atomic theory in the nineteen-twenties. Local and County histories are useful sources of information on these matters, but if one is not particularly adept at this kind of activity, a few words in the ear of an enthusiast in local history will usually produce a flood of useful information.

A TEACHER'S QUESTIONS

R. M. FYFE

Whether we consciously admit it or not, we are, as teachers, in the dominant position in the classroom: we are the "master" or "mistress". Do we accept this responsibility?

The class before us is a set of individual children, each of whom comes to us as he is. Firstly, then, we should ask ourselves what this means. Do we know what it is to be a child of this age? What are their feelings? What interests them? How do their minds' work? Secondly, we shall be helped in our meeting with them if we can discover what effect we have on them. If we understand this, we shall be able, consciously, to arouse a suitable mood in them, and stimulate their energies. When we are being sincere with ourselves, we never imagine that we are perfect: self criticism can help us to see aspects of our speech and mannerisms that can only distract the children; lack of self-confidence inhibits the infectious liveliness that enters naturally whenever an attempt is made to convey to others something that deeply interests us and that we are convinced is worthwhile. Thirdly, what have we to offer to the enquiring mind of the child? Can we create in the classroom a unity of endeavour, a *working together*, in order to arrive at some cogent discovery, or towards the gaining of mastery: that fullness of understanding which inevitably gives satisfaction?

Having such questions, albeit unanswered, clearly in mind, we shall have gained freedom to engage on the practical task of deciding what to attempt in tomorrow's lesson with such and such a class. At this moment our aim is different. What we now do is to rethink the mathematics involved, to assure our own mastery of the subject matter. What does this particular fact mean? What mental structure is needed that it may become known, and what facts, already known, relate to it? How can the method of dealing with it, or the examples introduced in conjunction with it, lead forward towards landmarks in the future work of the course?

From the humility of doubt and self-quesioning can arise a moment of creative boldness in which we ask ourselves (incredulous at first): "Is it possible that the 'best' way of dealing with this topic has not yet been discovered?" Thus encouraged, we set to work to think of a dynamic situation, one that is sure to interest the children, and that also is pregnant with the mathematics which interests us. If the two lines of interest become fused, the aim of the lesson will be achieved.

Tomorrow comes, and we enter the classroom with confidence. We meet the children and present our "situation" and watch their reactions. In conceiving it, we could see only what our conditioned, adult minds enabled us to see. The children will see it differently and they will see *more*. If we have achieved a feeling of self-mastery, we shall have our energies freed of anxiety about the success or

failure of the lesson: thus we can watch, alert, to see what happens, and listen carefully to the things the children say. By this means, we learn from them.

Developments which arise during the lesson may be well worth pursuing. We should be ready and willing, if it seems desirable, to change direction immediately, either to deal with a point we had supposed known, but which proves to be obscure, or to explore a new avenue which we perhaps did not suspect existed, or did not dream would arise in this connection. Ability to achieve a successful *volte face* shows that we have accepted that teaching is an art.

The bell rings: the class departs. Now, or hereafter, we relive the experience of the lesson in our minds. What happened? Where are we? What original contribution did the children themselves make? What could, with advantage, have been done differently? What points arose that were not adequately dealt with? Where do we go from here?

FORTHCOMING MEETINGS

By the time this bulletin is in print, the following Spring Meetings will have taken place:

Feb. 8th. At Nunnery Wood Secondary School, WORCESTER.
Day Conference: *Mathematics in Primary and Secondary Schools.*

March 8th. At Greenford County Grammar School, Greenford.
Second MIDDLESEX BRANCH Meeting: *The Secondary School Syllabus.*

March 14-16. At Redman's House Hotel, BLACKPOOL.
Week-end Conference: *Secondary School Mathematics—Content and Method.*

The following meetings are also being organised, and members are invited to support them, and to offer assistance:

May 15th. Second DONCASTER BRANCH Meeting: *Standardisation in the Primary School.* Particulars from R. W. Shaw, 23 Pipering Lane, Scawthorpe, Doncaster.

June 7th. Day Conference at CAMBRIDGE. *Mathematics for Pupils aged* 11-15.
a.m. Demonstration lessons and discussion: Teaching of Fractions, J. V. Trivett. Geometry, I. Harris.
p.m. Symposium: Broadening the Modern School syllabus, and the place of Aids in Modern and Grammar School teaching. D. T. Moore, J. V. Trivett, Mrs. R. M. Fyfe.
Exhibition:

Particulars from the Secretary and from Mr. P. Carpenter, Cambridge Institute of Education, 2 Brookside, Cambridge.

June 21st. Day Conference at CHICHESTER. *Mathematics in Secondary Modern Schools.* Details from J. V. Trivett, 16 Southwood Drive, Bristol 9.

Dates to be confirmed: Particulars will appear in the next bulletin.

Oct. 11th. OXFORD: *Spatial Relationships.*

Nov. 14/15th. CARDIFF: *Mathematics for* 11-15 *year olds.*

The SECRETARY and the EDITOR are urgently in need of assistance, preferably from members who live near them.

CORRESPONDENCE

Dear Sir,

Having taught in Primary Schools, in London, Berkshire and Kent, for many years, I am now in a position to look back and wish that a system such as the Cuisenaire had been introduced before.

How much easier and more interesting it would have been to teach arithmetic with the coloured rods rather than with the aid of sticks, beads and dominoes. How much quicker the children would have comprehended the fundamental facts of arithmetic.

Last year, during the summer holidays, I was introduced to Number Colour Rods and was so impressed with the simplicity and yet far reaching possibilities of the system that I sent for a set of rods and the book *Numbers in Colour*.

The class to which I returned in September, 1956, was a so-called 'backward' one consisting of 29 boys and girls aged 7-8. Their backwardness was due to a number of reasons: illness, and therefore absence during Infant School days; changes of district and therefore change of school and a few children with really poor mentality.

I found that many of them were disinterested and not ready to make any effort to work until I gave them, in turn, my set of rods with which to play.

Soon two more sets of rods were given to the class by the Headmaster who was intrigued with the new apparatus and interested in the book *Numbers in Colour*.

Having more sets of rods enabled more children to have periods of play with them each morning and I soon found the children sorting the colours, building and making patterns. I found myself learning with the children to *write* the patterns in colours.

Then counting the white rod as one we found the value of each of the others up to the orange 10, and within a week all the children were familiar with the colours and values of all the rods and we played many games.

Suddenly one boy called out excitedly, "I've made a sum!" From them onwards sums involving all four rules were *discovered*, literally discovered by the children and the enthusiasm during Arithmetic periods grew daily. This enthusiasm spread to other lessons and I was thrilled at the end of the school year with the tremendous progress made in all subjects and I am sure that it was the Number Rods that had awakened new interest.

I hope that many Primary School teachers will experiment with the new way of teaching Arithmetic and will have the happy, encouraging results with which I have met.

G. M. HOLLETT.

A NEWTON STAMP

Isaac Newton appears on a new 18 franc stamp in France. The stamp is one of a series of seven bearing portraits of outstanding non-French contributors to post-Renaissance western civilization. The other six included are Copernicus, Michael Angelo, Cervantes, Rembrandt, Mozart and Goethe.

LONDON HEAD TEACHERS' EXHIBITION

At the London Head Teachers' Association's Educational Exhibition at the Central Hall, Westminster, in October, mathematics had a special place. The main part of the exhibition was presented by commercial firms and covered all subjects, but one room was set apart for a display of mathematical material made by teachers and children. It is not perhaps reasonable to compare the material shown here with the professional displays elsewhere in the exhibition, but seeing the two in such close proximity one could not fail to notice how far more immediately attractive the professional work was, even though further investigation often showed it to be trivial and devoid of ideas that matter. The teachers' work was often much richer in ideas but its full potential impact was often not made because it lacked the final polish that one could see in the adjoining rooms.

One must distinguish sharply between aids made by pupils for their own instruction (and never intended for public display) and aids made by teachers for display to their pupils, but is professional finish so far beyond the powers of a classroom teacher? Should commercial art form part of the course at a training college? Before answering let us remember how children are surrounded by attractive advertisements designed especially to appeal to them. Can school science be made as attractive as science fiction, and can mathematical configurations be made as appealing as advertising doodles on commercial television? Certainly mathematics would benefit if teachers could put across their ideas with the verve and polish which characterise the public relations of the best business firms—and if this is 'salesmanship' then the classroom badly needs it.

One of the many exhibits evoked a certain nostalgia. What a fine teaching aid the old-fashioned bus ticket was! Gaily coloured, firm to hold and bravely bearing a bodly printed number for all to read. Can the present sorry roll of flimsy paper, anæmically printed, be called progress?

It was a pleasure to see four demonstration lessons on the programme, covering an age range from Infants to Secondary. With so much going on we were, unfortunately, only able to watch one lesson all through. This was the introductory lesson on trigonometry given by Mr. Blackman, of Forest Hill Secondary Boys' School, who used an effective display of triangles in coloured paper, a simple height finder and a large triangle made with a length of rope which he had tied to an 'inaccessible' lamp bracket before the lesson began. The conversation of the teachers as they left the hall showed how the lesson had won their approval.

Our congratulations are due to Mr. G. W. Gray, an L.C.C. Staff Inspector, for arranging the demonstrations and the display of school work, and to all the teachers who helped to make them both a success.

Mrs. Fyfe proposes to arrange discussions among small groups at her flat on Wednesday evenings from 7.30 to 10 p.m. Will members who would like to be invited please write to her. Suggestions for subjects to be discussed will also be welcomed.

The Editor also proposes to arrange a 'Working Party' which will meet at his house from time to time to work on projects of interest to the Association, such as the production of models and demonstration apparatus, films and articles for the bulletin. Anyone interested should write to him, or ring SYDenham 4322.

THE TEACHING OF NUMBER IN THE PRIMARY SCHOOL

The University of Nottingham Institute of Education has arranged a course of five lecture discussions on The Teaching of Number in the Primary School. The talks, and speakers arranged are:

Saturday, January 25th: Mr. Nathan Isaacs, *Professor Piaget's New Light on Children's Ideas of Number.*

Monday, February 17th: Miss J. Clarkson, *The Cuisenaire System of Number Teaching.*

Monday, March 17th at 5.30 p.m.: Miss H. Davis, *The Abacus and Number Concepts of the Child.*

Monday, May 5th at 5.30 p.m.: Miss Winifred Ferrier, *Real Life Number Scheme.*

Monday, June 2nd at 5.30 p.m.: Mr. H. Shaw, *Structural Arithmetic.*

Particulars of the course are obtainable from The Director, Institute of Education, Derby Road, Nottingham; but readers are advised that at the time of writing the course is already oversubscribed and we understand that no more places are available.

THE MODERN SCHOOL AT OXFORD

A one-day conference on the practical problems of teaching mathematics in the Modern School was organised by the Oxford Institute of Education on Thursday, December 5th, 1957. Mr. A. P. Rollett, H.M. Staff Inspector of Mathematics, opened the conference, pointing out that often the A stream is not stretched enough and that the C stream attempt work which is too difficult for them. The staff available had to be used to best advantage and there should be a specialist responsible for all the mathematics in the school. Each class should be taken by only one master. It seems that too much attention is being given to arithmetic and that elementary algebra, geometry, trigonometry and technical drawing can all be brought into a syllabus of mathematics. The subject should be joined up with Science, Geography, Handicraft, Domestic Science, etc., and the children should be encouraged to break down the division into watertight compartments. The ill-prepared teacher can often create the wrong attitude. Supplementary courses were mentioned and Mr. Rollett hoped that more teachers would take them.

Mr. D. Smith, Headmaster of the County Secondary School, Mexborough, gave a detailed talk on the organisation of the teaching of mathematics in the Secondary Modern School, and afterwards Mr. Harold Fletcher and the Revd. H. C. Babb spoke on the Content and Applications of the course. Both speakers pointed out the vital necessity to make the subject real and alive, and suggested schemes such as Card Assignments to make the pupils think and discuss.

COMPUTERS FOR SCHOOLS

In the May, 1957, issue of this bulletin we reported the work which the Wolverhampton and Staffordshire College of Technology were doing with computers. Mr. R. Woolbridge, Senior Lecturer in Mathematics at the College writes further:

Our first programme of lectures and demonstrations for the mathematics staff

and Sixth Form mathematics pupils of local schools took place on December 4th, 1957. The programme was designed to introduce pupils to calculating instruments and machines and to give them the opportunity of seeing the type of machine they will probably use in industry or at University on leaving school.

The course consisted of

(1) A lecture—The History of Calculating Instruments and Machines illustrated by lantern slides and photographs and the machines held at the college.

(2) A showing of films of five modern high-speed computers in use in the country today.

(3) A demonstration of the Dekatron Computer installed at the college.

(4) An exhibition of desk calculators by the leading manufacturers in the country.

The response of schools to the venture was most encouraging. In fact the demand for places far exceeded the accommodation we had available, so that we had to restrict numbers. Pupils came from schools in places as far afield as Birmingham and Malvern.

NEW PUBLICATIONS

In recent months *Mathematical Pie* has made available more material which may be of assistance to teachers of mathematics.

Recurrence Relations

To encourage 'off syllabus' reading by Senior students and also to foster minor research a series of 16-20 pp. booklets is in preparation. The first one, *Recurrence Relations*, is now available, price 1/- (postage extra according to number of copies). These booklets are printed in style and size similar to *Mathematical Pie* and are bound in an attractive two-colour cover. They should appeal to anyone interested in mathematics and having a slight knowledge of geometric progressions. It is hoped that teachers will encourage senior pupils to secure their personal copies of the booklet in order that it may be possible to continue to keep the price of the booklet at the present level. Other titles in the series will be published from time to time if the demand is sufficient.

Wall Charts

In the series of Wall Charts two are now available, *Beginnings of Astronomy No.* 1 (as Maths. Pie No. 17) and *Time Chart No.* 7 (as Maths. Pie No. 19). These charts are 21″ × 15″ mounted on thick mill board, punched, eyeletted and strung. The price (including home postage) is 7s. 6d. (overseas 2s. 6d. extra) whilst unmounted copies are available, price 4s. (overseas 1s. extra). There will be additions to the series in the future.

Nail Boards

Class sets of nail boards for elastic band geometry and teaching other forms of spatial relationships can now be obtained, price 3s. each board (postage extra). These boards are 9″ square (36 nails) and are finished in pastel shades. Quotations for other size nail boards and for other teaching aids in wood can be obtained on request if some indication is given of approximate requirements.

Mathematical Pie

Many subscribers have asked for copies of early issues which have been out

of print for some time but the heavy cost of typesetting has prevented this. A cheap method of reproduction of small quantities has however been found and copies of No. 1 are now available at 2d. per copy (postage 3d. if less than 6 copies).

Film-strips

The History of Numerals, Part II, is now available. The two parts together deal with the gradual development of our present methods of writing numerals, beginning with 'concrete' number and by means of diagrams, maps, time charts and photographs of the various numeral systems show their evolution into the abstract.

Part I (27 frames, black and white, price 15s.) deals with the primitive numeral systems and their methods of writing, and then later the story as far as the Egyptian and Babylonian systems.

Part II (37 frames, black and white, 15s.) continues with the Grecian and Roman systems and, via the Arabic, Hindu, and other systems, shows the gradual emergence of the present method of writing our numerals.

Another strip now available is *Alice in Numberland* (16s. 6d.) by Canon D. B. Eperson, M.A., a mathematical fantasia on themes by Lewis Carroll.

History of Calculation, Part II, which will deal with electronic computors is also in the course of preparation. In connection with the strip a search is being made to obtain photographs of the following illustrations:

(1) A Nautical Almanac. (2) A National Calculator. (3) Jacquard Loom, and if any members of the Association can help, Canon Eperson (Bishop Otter Training College) would be pleased to have information concerning them.

The script of a further film-strip *Elementary Solid Geometry and Trigonometry* is also being developed. If any members of the Association have ideas on this topic their co-operation would be welcomed.

All enquiries concerning these developments should be made to R. H. Collins, 97 Chequer Road, Doncaster.

CHANGE FOR A POUND

In how many ways can you give change for a pound using the normal coins ¼d., ½d., 1d., 3d., 6d., 1s., 2s., 2/6d., or a 10s. note? Mr. R. G. Mills, of Elliott Brothers, has given the answer. He reports:

The result was computed to be 324,946,182 on the Elliott 402 computer. First of all the computer tried every solution at the rate of about 40 per second; this clearly was too slow and would have actually taken *about* 2,200 *hours*. During the time the computer was working on this I noticed that for every solution including n pence, but not including smaller denominations, there are $(n+1)^2$ solutions including smaller denominations. Using this fact cut the time down to 1½ *hours*, the rate of finding solutions in whole numbers of pence being the same.

The basic programme was adapted from a sub-programme originally developed for the optimum cutting of glass sheets, which may also be useful for cutting steel strip and bar, paper rolls, etc. In these cases, both the "£1" and the smaller denominations are continually changing depending on the orders required and the production output. For example, in the production of rolls of paper, edge flaws may occur which effectively reduce the width of the roll, which must then be sliced up to meet all sorts of customers' newsprint requirements.

WEEK-END CONFERENCE AT RICHMOND

DOES THE TEACHING OF MATHEMATICS PRESENT A PROBLEM IN THE SECONDARY MODERN SCHOOL?

On November 8th, at the Palace House Hotel, Richmond, twenty teachers met to discuss some of the general problems of teaching mathematics at the secondary level.

There were some who had come to speak and some who thought they had come to listen, but before the week-end was over we had each contributed to the discussion, and we had all listened. It was good, and refreshing, to hear the comments and criticisms and problems of all; encouraging if one's methods are commented on favourably, but useful if the criticism is adverse. To meet one's colleagues helps to dispose of the unpleasant feeling that one is working in a vacuum.

The meeting was opened by Mr. J. V. Trivett, of Bristol, who spoke on the "New Look" in the teaching of Mathematics. He pointed out how the teaching of Art had been given a new look, and that Mathematics must follow suit. He said that the traditional methods of teaching this subject had not proved successful, and that a new approach was necessary. Dr. Gattegno's definition of mathematics was quoted: "Variations upon virtual actions."

Mr. Trivett went on to say that, rather than impose a set of rules, or a line of action that the child must follow, it was better to place the child in a mathematical situation where his dynamic thinking would lead him along paths of his own choosing, where he would lead and the teacher follow.

This theme was enlarged upon and we were later shown many and varied examples of work done by his pupils. This included enlarging by projection methods, work on intersecting circles, approximation to areas, investigations of number series, pupil's own research into equivalences and transposed formulæ, magic squares, 'envelopes' by stitching and paper folding.

"Our job is to train children, not for the society of to-day, but for that of twenty years hence," Mr. Trivett concluded.

Mrs. Fyfe then introduced us to Cuisenaire rods which, she said, gave children meaningful practice in real number situations. She showed us how to use the material to show sums and differences, products, and how to deal with fractions in a realistic way. She also indicated some uses for the beginnings of algebra and for dealing with surface area, volumes and perimeters and properties of cuboids.

Miss Giuseppi then demonstrated Geo-boards on which with elastic bands, we all made shapes, investigated equal areas, symmetry and parallels, and all attempted a ten-minute piece of research.

Mr. Harris spoke on *Modern Schools and Geometry*. He questioned the validity of the "utilitarian" aim in the traditional method of teaching the subject. The new approach he advocated was one of "looking into mathematics to see what it can do." It was self-generating and a new technique of exploration.

He also defined the "new algebra" as operations on operations and the "new arithmetic" as operations on numbers. The study of geometry must include a more positive and dynamic "looking into geometry" which took the effects of movement into account. He stressed that oral discussion was vital and gave the incentive for rigour in communication. He also told us how important and rewarding it is to listen to what the children say.

Mr. Collins showed us his film-strip *History of Calculation*. He also stressed that the aim is not to "teach a set of tricks" but to start the child thinking for himself.

Much time was given to general discussion and questions, and usually led to the importance of getting the children to be "thinking dynamically" all the time. At the end, everyone present contributed his or her own criticisms which, as Miss Giuseppi pointed out, were both a helpful and necessary part in the breaking of any new ground.

E. MASON.

DAY CONFERENCE FOR MATHEMATICS TEACHERS OF MIDDLESEX AND N.W. LONDON

An open meeting was held on Saturday, November 30th, 1957, at Greenford County Grammar School, Greenford. It was attended by 80 teachers, of whom one half were already members of the Association. The Chairman was Mr. J. Wilkinson, Education Officer for the Borough of Ealing.

The morning session began with a lecture, *How Can Geometry Be Presented Dynamically?* by Mr. Ian Harris, Senior Mathematics master of Dartford Grammar School. He suggested that the question *What is Geometry?* should be put by teachers to their pupils and to themselves. For him, it was a study of invariants under certain transformations. In the Euclidean field these transformations consisted of translation, rotation and homography, and he gave vivid illustrations which convinced many present that the conventional approach to elementary geometrical properties, by suppressing consideration of the possibilities of movement, seriously limited geometrical understanding.

Mr. Harris included in his lecture a brief sketch of the history of geometry, stressing the unfortunate stultification that had resulted from Euclid's great formal achievement. He also drew a useful distinction between the attitude of the pure and the applied geometer and suggested that teachers might confuse their pupils by calling indiscriminately on the methods of both, without making the contrast clear. The approach of the pure geometer with which teachers were concerned, was to pass from concrete reality, through the model, to the abstraction.

In answer to a question, Mr. Harris discussed the contrast between the conventional or 'static' proof—a sequence of facts, and the more convincing 'dynamic' proof—a sequence of events; he admitted the greater difficulty of communication involved in the latter and suggested a compromise consisting of dynamic thinking combined with an ability to express in static terms.

In the absence of Mr. J. F. Davis, Headmaster of Stanhope Secondary School, a discussion on *The Arithmetic Content of the Syllabus* was introduced by Mrs. Fyfe. In her remarks she suggested that because there were so many failures in Mathematics at Ordinary Level, even among the top 10-20% of the school population, the conventional methods of drilling in techniques had been proved inefficient. She had decided she would never teach a technique unless the mathematics involved were first understood by the pupils: a serious obstacle was the excessive impregnation of all branches of mathematics with complex arithmetical calculations. In view of the teacher's responsibility to the pupils, and the existing condition of the syllabus, it was recommended that at least the first three years could be used for the gaining of mathematical insight, and that intensive practice of techniques could be undertaken in the last two years.

A wide ranging discussion followed, touching on criticism of examination questions, the utilitarian aspect of the Arithmetic syllabus, and points of methodology. There were some irrelevances, but other, unanswered, questions gave mental stimulation.

During the lunch break, members of the conference were able to see an attractive exhibition of models, brought by Mr. L. A. Swinden, and a display of pupils' work.

In introducing her demonstration of *The Use of Learning Aids* Mrs. Fyfe said that she hoped it had already been made clear that one thing the active members of the Association had in common was their belief in a *learner-centred* approach; that teachers were themselves learning aids. Mathematics exists in the mind, and aids are symbols which act as catalysts of mathematical thought.

Mrs. Fyfe showed how she used the Cuisenaire coloured rods to evoke algebraic statements of linear relation by using the initial letters of the colours as recognition symbols. It was made very clear that by presenting the pupils with a visual representation of algebraic equality and displaying it to them in different ways (by reversal, inversion and transposition of the rods, together and separately) the various equivalent forms of each relation could be made obvious. Statements involving subtraction followed at once, and powers and multiplication could be demonstrated by using an appropriate visual convention. It had also been successfully used for teaching permutations to young pupils.

The uses of Geo-boards were then demonstrated. All the angle properties of circles could be derived from the boards, and many area properties, including Pythagoras's theorem. Mrs. Fyfe stressed that only boards with a limited number of pins (not more than 25 for the square lattics) were recommended, so that children could gain the satisfaction of exhausting the possibilities of the topic under discussion.

She also pointed to examples of the uses that could be made of dynamic drawings of geometrical situations, mentioning the 'film-strip' technique for showing moving circles.

The discussion made it clear that those present had found the day stimulating, and would like to form themselves into a Middlesex group within the Association. The date fixed for the next meeting was Saturday, March 8th, and Mrs. Fyfe was asked to again hold it at Greenford School. Thanks were expressed to the Chairman and the organisers.

D. WHEELER.

WEST RIDING GROUP

At a meeting of 22 members at the Doncaster Technical College on January 18th it was decided to form a West Riding Group of the Association. It was agreed that future meetings should be held once a term at schools throughout the area—the next meeting being at the new Don Valley High School, Scawthorpe, Doncaster.

In the afternoon session of the meeting, Mr. Collins showed a short film on animated Geometry and warned against the easy assumption that a class would recognise in it a theorem already learnt in a static context. Also, Mr. Pointon, of Martin Primary School, gave a talk on some of the difficulties encountered in the early arithmetic syllabus. This latter topic will be followed up in the next meeting.

when the standardisation of content of Primary School Mathematics will be discussed from the points of view of the Primary School and Selective and Non-selective Secondary Schools. Mr. Atkin also will talk on Models in Solid Trigonometry and will show some of his own models.

Members in the area who have not already been notified of the formation of the Group are invited to write to the Secretary, Mr. R. W. Shaw, 23 Pipering Lane, Scawthorpe, Doncaster, for further details. R. W. S.

NORTH-WESTERN GROUP MEETING

There was an attendance of about sixty at the Manchester University School of Education on Saturday, February 1st, when Mr. J. Peskett gave a talk and demonstration on *Models and Methods of Making Them.*

In the course of an informative and interesting lecture he showed that models need not be complicated or expensive in order to achieve their purpose. The essential purpose of making a model was to put over the particular point which one wished to make. By actual demonstration, Mr. Peskett showed that it was possible for simply-made models to illustrate quite complex principles. In the course of the lecture he displayed a wide variety of materials which he had used and gave a wealth of practical details on the methods of working with these materials.

There was a representative display of models on show, and the meeting concluded with a brief discussion on teaching method in elementary algebra. As a result of this the group were given their homework: "To consider practical means of demonstrating and teaching elementary algebraic processes", and it is hoped to hear the findings of individual members and have a discussion on this topic as part of the next Manchester meeting which will be held in the Autumn term.

The next meeting of the Group is at Preston on Saturday, May 3rd, when Mr. R. H. Collins will speak. Details obtainable from Mr. C. Birtwistle, 1 Meredith Street, Nelson, Lancs.

3-D

Working on the Film Unit camera Mr. T. J. Fletcher has completed three scenes of stereoscopic material and has made them up into a demonstration film lasting five or six minutes. As far as we know this is the first animated mathematical film ever to be made in 3-D. Stereoscopic films are only justified when all other methods of presentation fail, and the immense labour involved in producing them is not to be considered until one wishes to present a piece of three dimensional geometry which cannot be shown adequately by a movable model. The film which has just been completed shows three different modes of electromagnetic oscillation in a resonant cavity. The lines of magnetic force are shown in green and the lines of electric force in red, so the film is not only the first stereoscopic mathematical film to be made, it is also the first piece of mathematical animation to be seen in colour in this country.

The Nuffield Foundation, who previously gave a grant for the purchase of the animation camera, have given a further grant of £50 to enable copies of this and other material to be made later, to be made and distributed for experimental showings. We give them our most grateful thanks for this generous assistance.

BOOK REVIEWS

Know Your Maths. Book 4. Keith and Martindale. Blackie, 1957. 8s. 3d. with answers, 7s. 9d. without answers.

As with the earlier books of this series, much care and attention to detail have clearly gone into the writing of this book; but, alas, they seem to me very largely misdirected.

The arbitrary division of the subject into disconnected branches is a very severe fault, which three examples may serve to illustrate.

(1) The sections on formulæ and graphs are separated—there is no attempt to relate together the table, the verbal statement, the graph and the formula, as different ways of showing relationships.

(2) The graph of squares is presented ready-made in the section on graphs as a 'Ready Reckoner' type, its sole use being apparently the reading-off of square roots.

(3) The section on mensuration is divided into sub-sections 'Plane Areas', 'Surface Areas', and 'Volumes', so that an old and interesting friend like the cylinder appears in separate and unrelated guises—a pawn, as it were, in the game of arid classification of entities.

An even deeper revulsion is felt, by anyone who has seen children's joy in mathematics, towards the authoritarian flavour and persuasive reasoning (mostly spurious) which pervade the book. For instance, to encourage him to tackle the set of definitions and exercises on the circle properties, the reader is told—"Mathematics is one of the best school subjects for training pupils to think things out for themselves. In the section which follows, practice is therefore given in this kind of work. The exercises you will be given may seem far removed from urgent everyday problems, but the same kind of reasoning is involved. . . ." What false propaganda is this? Couched, in such terms, how many 'average' fifteen-year-olds would trouble to read it? Perhaps it is intended for the non-mathematical teacher—the exercises which follow seem designed to deaden thought and not invite it.

Some of the exercises are better than these, and the arithmetic sections could be genuinely useful to the 'average pupil' for whom the book is intended. While respecting the care and work involved, however, here is one reviewer who can not recommend this book.

M. J. Meetham.

Practical Arithmetic For Boys—R. E. Harris. Macmillan & Co. Ltd.

This book does for boys what 'Practical Arithmetic for Girls' did three years ago; it is well produced with clear print and appealing line drawings. For those pupils who are anxious to obtain practice in simple calculations concerning travel, mortgages, wages, motoring, home decoration, crafts and so on, the book should have an appeal. But it does mean that the boy will have to read well. In fact, the standard of reading implied would appear to be far above that required for understanding the arithmetic. The conclusion from this is that such a book will only appeal to those who really decide to make a big effort.

Used by a teacher, however, the information within may provide a good foundation for lessons made less pedestrian than the printed form. Real maps, timetables, etc., should augment those in the book and the examples quoted used as a

basis for real enquiry. Very little of the subject-content will ever appear to the boy at work as textbook exercises and the sooner he can have real experience 'on location,' real or artificial, the better.

At the end of the book the fundamental arithmetic processes are set out, with rules. It is not convincing that this will do other than temporarily remind even though the reader brings to it much incentive to work through the examples and generalise from the particular cases quoted.

In short, the book is a sincere, well-produced attempt at recognising the reality of the working world and the need for simple arithmetic to cope with it, but presented inevitably in an unreal form.

J. V. TRIVETT.

AN INTRODUCTION TO APPLIED MATHEMATICS. G. W. Walters. Macmillan & Co. Pp. 157. Price 6s. 0d. (1957).

A textbook for 'O' level examinations, this volume appears little different from others which cover this same field. It is a textbook geared to a course of mathematics which includes no calculus. The author attempts to keep the sequence of mathematical principles used in conformity with a conservative arrangement of the mathematics syllabus in which topics such as the cosine rule come late in the course.

It is primarily concerned to present a series of examples based on the formulæ of applied mathematics, carefully graded so that initial success is assured and obstacles and complexities are introduced gradually. A teacher who believes that the ideas of mechanics are physical and based on experiment will have to construct his own experimental instructions. The book supplies only brief explanations of the derivation of formula and their application to examination problems. It is customary nowadays, to include some historical and biographical detail in a course to 'O' level. Apart from one brief reference to Hooke's anagram, such references are omitted.

The order of topics is usual, except that the use of the cosine law in finding resultants of two forces is put in the middle of the book although resultants of parallel forces are treated geometrically in the first chapter. A proof of the cosine rule applied to the diagonal of a parallelogram is given and in this form the formula has the unfamiliar guise:

$$R^2 = P^2 + Q^2 + 2PQ\cos a$$

It seems a pity that for the sake of unity the author did not apply the ordinary formula used in trigonometry. The absence of examples on the graphical treatment of variable acceleration will consolidate the all too common fallacy with which students enter advanced courses that the equations of uniform motion have a universal application.

As an efficient aid to G.C.E. success this book is to be commended but as a stage in the development of an understanding of the principles of mechanics, it will require a great deal of supplementing. C. H.

MATHEMATICAL NOTES. C. F. G. MacDermott. Blackie. pp. 60. 3s. 6d.

When setting out to review any mathematical booklet intended to be used by students it would appear to be reasonable to begin by considering it against the background of mathematics teaching and to assess the value of the book in terms of a possible contribution to the work of the teacher. But in fairness to the author

this should not be done in the case of this booklet for it adds nothing in this direction nor indeed could there have been any intention to do this.

The contents comprise a collection of facts and formulæ divided up under the headings, Algebra, Analytical Geometry, Calculus, Geometry, Trigonometry, Statics, Hydrostatics, and Dynamics, with some proofs or hints of proofs of this bookwork most required for 'A' Level Mathematics.

It was obviously designed to have its main use immediately prior to any examination, but its value can then only depend on whether or not the teacher is convinced that being able to learn passages of bookwork by rote is a desirable part of mathematics teaching. This reviewer holds that this is not so and in any case believes that the best notebook for revision of this sort is the one compiled by each pupil in the course of his 'A' Level work.

The only part a printed booklet like this can play in revision is when it is provided with a few exercises for the student to work through at the end of each item or chapter. This is not the case here.

R. H. COLLINS.

BASIC MATHEMATICS FOR RADIO AND ELECTRONICS. F. M. Colebrook and J. W. Head. pp. 359. Iliffe. 17s. 6d.

This is a new edition of a book by Colebrook, previously published in 1946 under the title *Basic Mathematics for Radio Students*, with two additional chapters by Head. It gives a survey of the basic principles of those branches of mathematics which are of constant application in radio and electronics, starting from the very beginnings of algebra and working through to complex numbers, differential and integral calculus, Heaviside's operational methods and two by two matrices.

It is a difficult task to compress so much into just over 300 pages and a true beginner would find the pace a little fast at times,and he would find the shortage of numerical examples something of a handicap in the early chapters; but a reader for whom the first two chapters were merely revision would get a flying start into the work on complex numbers and infinite series and should find that the book meets many of his needs.

The treatment of negative numbers is probably self-consistent, but many mathematicians would (as is recognised in a footnote) find strong objections to it. On the other hand, complex numbers are done very well, and the view of them as operators on vectors is clearly explained and consistently applied throughout. If the author had treated directed numbers as operators in a similar way there would have been much less to criticise.

The chapter on Heaviside's technique starts by demanding "an act of faith in Heaviside and his assumptions," and informs the reader that "at this stage we can think of Heaviside's technique as an extraordinarily powerful means of enlightened guesswork as to the behaviour of electrical circuits." This type of approach does nothing to further the mutual esteem in which mathematicians and electrical engineers should hold one another; it is quite unjustified and completely wrong. This chapter needs complete revision. The final chapter starts with a useful introduction to four-terminal network theory, and goes on to explain, very briefly, a number of topics which the reader might well wish to follow up in greater detail later on.

In this book there is much to praise, and the success of the earlier edition indicates its qualities; it succeeds as a formal statement of formal mathematics but it

displays many of the present shortcomings of technical teaching with the utmost clarity. The authors are practical men and they claim to be writing for other practical men. The dust-cover claims that "the authors' lively style has vitalised and humanised what can so easily be made a dull subject." In my view this is exactly what they have failed to do. Occasional jocular remarks may lighten the task for a mature reader, but they do not humanise the subject when all through the book the method of presentation is the traditional 'pure' one—an abstract piece of mathematics, followed perhaps by an application. There is no attempt to show how the abstract ideas were so often suggested by previous practical experience, there is no attempt to introduce abstract theories in terms of concrete examples, and there is no attempt to show the reader how he can become a pure mathematician himself by the correct appraisal of a practical situation. We badly need a new and a truly practical approach—an approach showing how abstract ideas arise from the study of practical problems by practical men—but unfortunately this rather conventional book does not provide it.

T. J. F.

RECHERCHES SUR LA COMPRÉHENSION DES RÈGLES ALGÉBRIQUES CHEZ L'ENFANT, Lydia Müller, Delachaux et Niestlé, Neuchatel et Paris, 1957. pp. 242. Frs.8.

This is a thesis written under the direction of Piaget, who has also written the preface. As is well known, there is in Geneva a school of psychology which is investigating the genesis of mathematical ideas. Piaget himself has done much to clarify the formation of the ideas of number, quantity, space, time, motion, and chance, using young children for the most part as his subjects. Some of his pupils have worked with adolescents.

The present thesis follows the same pattern as those which have been published over the last ten years, guided by the ideas of Piaget as contained in his theories or in his method of study. The theory states that the whole of mental development is from sensory-motor experience to operational thinking through intuitive and practical operations, the complete reversibility of thought being one of the criteria most stressed. The method consists in placing the subjects in actual situations which exemplify the questions being studied, and measuring, by their success or failure, whether this or that level has been reached. Against this background, Miss Müller has studied the understanding of the rule of signs. Her thesis is based on the conception that this rule is similar to the negation of a negation, and that if a situation having this structure is created, the subject's ability to forecast the outcome will be equivalent to understanding of the rule. Such a situation is exemplified in particular by a car which can occupy two positions, one on either side of a road, and can move forwards and backwards in each.

Although the book is a study of the reactions of children and adolescents to a set of challenges, we cannot accept the underlying mathematical and psychological notions as justifying the meagre conclusions of this experimental work. Mathematically speaking, it is today somewhat naïve to seek in reality for a basis of the rule of signs, which is one of the possible mappings of E x E upon E where E is the set of real numbers. The real psychological difficulty arises from the fact that children are presented with the rule of signs when they are used only to thinking of arithmetical situations, whereas they need first to become accustomed to thinking

of correspondences which map sets of pairs of elements upon sets of elements. The meaning of the rule as being one of the possible choices of the correspondence then becomes natural.

Teachers who do not understand the true facts of the rule of signs will not get much help from the pedagogical conclusions of this thesis, which is itself based on a misunderstanding of the main topic investigated.

C. G.

BOOKS RECEIVED FOR REVIEW

An Introduction to Applied Mathematics, G. W. Walters. (Macmillan).
A Completion Trigonometry, W. J. Garlick & W. E. Wakefield. (Schofield & Simms).
Numerical Trigonometry, R. Walker. (Harrap).
Number Games and Projects, T. G. Monks. (Oliver & Boyd).
A Practical Calculus, A. G. Proudfoot. (Macmillan).
A New Reform Arithmetic, W. J. Kelly. (Macmillan).

A.T.A.M. LIBRARY

The following books, belonging either to the Association or to Mathematical Pie can be obtained from the Librarian provided postage from Library to borrower is returned with the book.

A History of Mathematics, Cajori.
Mathematics for the Million, Hogben.
Mathematical Recreations, Kraitchek.
The Great Mathematicians, Turnbull.
Mathematical Models, Cundy and Rollett.
Mathematical Wrinkles, Jones.
Mathematical Nuts, Jones.
Canterbury Puzzles, Dudney.
Modern Puzzles, Dudney.
Mathematical Snapshots, Steinhaus.
The Development of Mathematics, Bell.
General Elementary Mathematics, Hodge and Wood. Part 1 to 4.
The Theory of Numbers, B. W. Jones.

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